Topology

B. Math. II

Back Paper Examination

Instructions: All questions carry equal marks.

1. Define closue $\overline{A}$ of a set A in a topological space. If $f : X \to Y$ is continuous map, then is it true that $\overline{f(A)} = f(\overline{A})$? Justify your answer.

2. Let $f : X \to Y$ be a continuous map between two topological spaces. Prove that the graph $G(f) = \{(x, f(x)) \mid x \in X\}$ is a closed subspace of $X \times Y$. Is the converse true? Justify your answer.

3. Define open maps and closed maps between topological spaces. If X and Y are spaces, prove that the projection map $\pi_1 : X \times Y \to X$ is always an open map. Is it always a closed map? Justify your answer.

4. Define dense subset of a topological space. Prove that if X has a countable basis, then it has a countable dense subset.

5. Define connected and path connected spaces. Give an example of a connected space that is not path connected. Justify your answer.

6. Define regula topological space. Prove that a compact Hausdorff space is regular. Give an example of a compact space that is not regular and an example of a regular space that is not compact. Justify your answers.